Statistical Thermodynamics of Solutions of Optically Active Substances. II. Solubility of d- and l-Isomers in Optically Active Solvents

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It is expected theoretically that when *d*- and *l*-optical isomers are independently dissolved in an optically active liquid, the solubilities of *d*- and *l*-isomers may be different. To elucidate the special feature of this problem, the theoretical expression for the solubilities of *d*- and *l*-optical isomers is based on the model described in a previous paper[1]. Further a possibility of optical resolution is discussed.

Theory

Let us consider two kinds of systems, the one of which contains optically active liquid substance A and *l*-isomer of optically active molecule B (which is denoted by B_l) and the other contains A and *d*-isomer of optically active molecule B (which is denoted by B_d). For simplicity it is assumed that the two components are completely immiscible in the solid state and completely miscible in the liquid state.

The chemical potential of B_l molecule in crystal $\mu^K{}_{B_l}$ is a function of absolute temperature, T, alone while the chemical potential of the same molecule in solution $\mu^S{}_{B_l}$ is a function of, T, mole fraction of B_l, x_{B_l}, and interaction parameter between A and B_l molecules, ω_{AB_l}. For a saturated solution of B_l in A the chemical potential of B_l is equal to that in the crystal, then it follows that

$$\mu^K{}_{B_l}(T)=\mu^S{}_{B_l}(T, x_{B_l}, \omega_{AB_l}) \tag{1}$$

In a similar way for a saturated solution of B_d in the following expression is obtained

$$\mu^K{}_{B_d}(T)=\mu^S{}_{B_d}(T, x_{B_d}, \omega_{AB_d}) \tag{2}$$

where $\mu^K{}_{B_d}$ and $\mu^S{}_{B_d}$ are the chemical potentials of B_d molecule in the crystal and in the saturated solution respectively, and x_{B_d} is the mole fraction of B_d in the solution and ω_{AB_d} is an interaction parameter between A and B_d molecules.

As B_l and B_d molecules are mirror images for each other, the chemical potential of B_l molecule is equal to that of B_d molecule in the crystal at the same temperature. Then it follows from 1 and 2 that

$$\begin{aligned}\mu^K{}_{B_l}(T)=\mu^K{}_{B_d}(T)&=\mu^S{}_{B_l}(T, x_{B_l}, \omega_{AB_l})\\&=\mu^S{}_{B_d}(T, x_{B_d}, \omega_{AB_d})\end{aligned} \tag{3}$$

Since ω_{AB_l} and ω_{AB_d} have different values, x_{B_l} and x_{B_d} in each saturated solution have different values. It means that the solubility of B_d and B_l optical isomers differ from each other. In the following, explicit expressions for $\mu^S{}_{B_l}$ and $\mu^S{}_{B_d}$ will be derived. In deriving the theoretical expressions, the following assumptions are made:

(1) Molecule A, B_l and B_d are equal in size and each occupies a site of quasi-crystalline lattice and has $3z$ orientational freedoms.

(2) Molecule A is covered by z planes, one of which is composed of a element, and the others $(z-1)$ are composed of b elements. The a element is composed of three subelements 1, 2 and 3 and their arrangement is in the order 1, 2, 3 clockwise viewed from outside of the molecule.

(3) Molecule B (both B_l and B_d) is also covered by z planes, one of which is composed of a' element and the others $(z-1)$ are composed of b' elements. a' element is composed of three subelements $1'$, $2'$ and $3'$ and their arrangement is in the order $1', 2', 3'$ clockwise for B_l molecule and $1', 3', 2'$ clockwise for B_d molecule viewed from outside of the molecule.

(4) The configurational energy of the system is the sum of interaction energies of plane-plane pairs.

If the interaction energy of plane i and plane j is denoted by ω_{ij}, all possible kinds of ω_{ij} and their interaction energies become as follows.

A−A
$$\begin{aligned}&\omega_{bb},\ \omega_{ab},\ \omega_{aa_1}=\omega_{11}+2\omega_{23}\\&\omega_{aa_2}=\omega_{22}+2\omega_{13}\\&\omega_{aa_3}=\omega_{33}+2\omega_{12}\end{aligned}$$

$B-B_l$, $B-B_d$
$$\begin{aligned}&\omega_{b'b'},\ \omega_{a'b'},\ \omega_{a'a'_1}=\omega_{1'1'}+2\omega_{2'3'}\\&\omega_{a'a'_2}=\omega_{2'2'}+2\omega_{1'3'}\\&\omega_{a'a'_3}=\omega_{3'3'}+2\omega_{1'2'}\end{aligned}$$

$A-B_l$
$$\begin{aligned}&\omega_{bb'},\ \omega_{ab'},\ \omega_{a'b},\ \omega^{l}_{aa_1}=\omega_{11'}+\omega_{23'}+\omega_{32'}\\&\omega^{l}_{aa'_2}=\omega_{22'}+\omega_{13'}+\omega_{31'}\\&\omega^{l}_{aa'_3}=\omega_{33}+\omega_{12'}+\omega_{21'}\end{aligned}$$

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[1] K. Amaya, This Bulletin, 34, 1689 (1961).

$$\mathbf{A-B}_d \qquad \omega_{bb'},\ \omega_{ab'},\ \omega_{a'b},\ \omega^d_{aa'_1}=\omega_{11'}+\omega_{22'}+\omega_{33'}$$

$$\omega^d_{aa'_2}=\omega_{13'}+\omega_{21'}+\omega_{32'}$$

$$\omega^d_{aa'_3}=\omega_{12'}+\omega_{23'}+\omega_{31'}$$

The total configurational energy of the system is expressed as a sum of these energies. For a system composed of N_A molecules of A and N_{B_l} molecules of B_l, the configurational partition function Ω_{AB_l} is given by

$$\begin{aligned}\ln \Omega_{AB_l} - \ln\frac{(N_A+N_{B_l})!}{N_A!\,N_{B_l}!} - \ln(3z)^{N_A+N_{B_l}} - \frac{N_A\chi_A+N_{B_l}\chi_{B_l}}{kT} &= \frac{\langle W_{AB_l}(N_A, N_{B_l})\rangle}{kT} \\ &\quad -\frac{\langle W_{AB_l}(N_A, N_{B_l})\rangle^2-\langle W^2{}_{AB_l}(N_A, N_{B_l})\rangle}{2!\,(kT)^2} \\ &\quad -\cdots\cdots \end{aligned} \tag{4}$$

where

$$\chi_A=\frac{z}{2}\cdot\frac{1}{z^2}\left\{(z-1)^2\omega_{bb}+2(z-1)\omega_{ab}+\frac{1}{3}(\omega_{aa_1}+\omega_{aa_2}+\omega_{aa_3})\right\} \tag{5}$$

$$\chi_B=\frac{z}{2}\cdot\frac{1}{z^2}\left\{(z-1)^2\omega_{b'b'}+2(z-1)\omega_{a'b'}+\frac{1}{3}(\omega_{a'a'_1}+\omega_{a'a'_2}+\omega_{a'a'_3})\right\} \tag{6}$$

and $W_{AB_l}(N_A, N_{B_l})$ is the excess potential energy of the system and $\langle W_{AB_l}(N_A, N_{B_l})\rangle$ and $\langle W_{AB_l}{}^2(N_A, N_{B_l})\rangle$ denote the unweighted mean over all configurations of this quantity and of the square of it respectively. Then $\langle W_{AB_l}(N_A, N_{B_l})\rangle$ is given by

$$\begin{aligned}\langle W_{AB_l}(N_A, N_{B_l})\rangle &= (N_A+N_{B_l})\{(1-x_{B_l})^2\chi_A \\ &\quad +2x_{B_l}(1-x_{B_l})\chi_{AB_l}+x^2{}_{B_l}\chi_{B_l}\} \\ &\quad -N_A\chi_A-N_{B_l}\chi_{B_l} \\ &=(N_A+N_{B_l})x_{B_l}(1-x_{B_l})(2\chi_{AB_l}-\chi_A-\chi_{B_l})^{*}\end{aligned} \tag{7}$$

where

$$x_{B_l}=N_{B_l}/(N_A+N_{B_l})$$

$$\chi_{AB_l}=\frac{1}{z}\left\{(z-1)^2\omega_{bb'}+(z-1)(\omega_{ab'}+\omega_{a'b})+\frac{1}{3}(\omega^l_{aa'_1}+\omega^l_{aa'_2}+\omega^l_{aa'_3})\right\}$$

In evaluating $\langle W^2{}_{AB_l}(N_A, N_{B_l})\rangle$, the average value over most probable configurations is taken as an approximation, since it is very difficult to evaluate the exact value of this quantity averaged over all possible configurations. For most probable configurations, the number of plane-plane pairs for each kind of pair becomes as shown in Table I.

The average value over the above configurations becomes as follows with a procedure similar to that described in the previous paper[1].

$$\begin{aligned}&\langle \mathrm{W}_{AB_l}(N_A, N_{B_l})\rangle^2-\langle W^2{}_{AB_l}(N_A, N_{B_l})\rangle \\ &=-\frac{(N_A+N_{B_l})^2}{4z^2}\Big[(1-x_{B_l})^4\cdot\frac{1}{36}\{(\omega_{aa_1}-\omega_{aa_2})^2 \\ &\quad+(\omega_{aa_2}-\omega_{aa_3})^2+(\omega_{aa_3}-\omega_{aa1})^2\} \\ &\quad+4x^2{}_{B_l}(1-x_{B_l})^2\cdot\frac{1}{36}\{\omega^l_{aa'_1}-\omega^l_{aa'_2})^2 \\ &\quad+(\omega^l_{aa'_2}-\omega^l_{aa'_3})^2+(\omega^l_{aa'_3}-\omega^l_{aa'_1})^2\} \\ &\quad+x^4{}_{B_l}\cdot\frac{1}{36}\{(\omega_{a'a'_1}-\omega_{a'a'_2})^2+(\omega_{a'a'_2}-\omega_{a'a'_3})^2 \\ &\quad+(\omega_{a'a'_3}-\omega_{a'a'1})^2\}\Big]\end{aligned} \tag{8}$$

By substituting 8 into 4, the following expression for configurational partition function Ω_{AB_l} is obtained.

TABLE I

The kind of pairs		Number of pairs	
$A-A$	$b-b$	$(z-1)^2/z^2\cdot N_{AA}$	$N_{AA}=\frac{1}{2}(N_A+N_{B_l})z(1-x_{B_l})^2$
	$a-b$	$2(z-1)/z^2\cdot N_{AA}$	
	$a-a$	$1/z^2\cdot N_{AA}$	
B_l-B_l	$a'-b'$	$(-1)^2/z^2\cdot N_{B_lB_l}$	$N_{B_lB_l}=\frac{1}{2}(N_A+N_{B_l})zx^2{}_{B_l}$
	$a'-b'$	$2(z-1)/z^2\cdot N_{B_lB_l}$	
	$a'-a'$	$1/z^2\cdot N_{B_lB_l}$	
$A-B_l$	$b-b'$	$(z-1)^2/z^2\cdot N_{AB_l}$	$N_{AB_l}=(N_A+N_{B_l})z\cdot x_{B_l}(1-x_{B_l})$
	$a-b'$	$(z-1)/z^2\cdot N_{AB_l}$	
	$a'-b$	$(z-1)/z^2\cdot N_{AB_l}$	
	$a-a'$	$1/z^2\cdot N_{AB_l}$	

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* Mean cohesive energy of a B molecule in an imaginary liquid state.

$$\ln \Omega_{AB_l} = \ln \frac{(N_A+N_{B_l})!}{N_A!\,N_{B_l}!} + \ln (3z)^{N_A+N_{B_l}}$$
$$+\frac{N_A\chi_A+N_{B_l}\chi_{B_l}}{kT}\ \frac{(N_A+N_B)}{kT}(2\chi_{AB_l}$$
$$-\chi_A-\chi_{B_l})\,x_{B_l}(1-x_{B_l})$$
$$+\frac{1}{2(kT)^2}\frac{(N_A+N_{B_l})^2}{4z^2}\Big[(1-x_{B_l})^4\cdot\frac{1}{36}\{(\omega_{aa_1}$$
$$-\omega_{aa_3})^2+(\omega_{aa_2}-\omega_{aa_3})^2+(\omega_{aa_3}-\omega_{aa_1})^2\}$$
$$+4x^2{}_{B_l}(1-x_{B_l})^2\cdot\frac{1}{36}\{(\omega^{l}_{aa'_1}-\omega^{l}_{aa'_2})^2$$
$$+(\omega^{l}_{aa'_2}-\omega^{l}_{aa'_3})^2+(\omega^{l}_{aa'_3}-\omega^{l}_{aa'_1})^2\}$$
$$+x^4{}_{B_l}\cdot\frac{1}{36}\{(\omega_{a'a'_1}-\omega_{a'a'_2})^2+(\omega_{a'a'_2}-\omega_{a'a'_3})^2$$
$$+(\omega_{a'a'_3}-\omega_{a'a'_1})^2\}\Big] \qquad (9)$$

Then the chemical potential of B_l molecules $\mu^{S}{}_{B_l}$ can be obtained by partial differentiating $-kT\ln\Omega_{AB_l}$ with respect to N_{B_l}

$$\mu^{S}{}_{B_l}=-\frac{\partial kT\ln\Omega_{AB_l}}{\partial N_{B_l}}=kT\ln x_{B_l}-\chi_{B_l}$$
$$+(1-x_{B_l})^2(2\chi_{AB_l}-\chi_A-\chi_{B_l})$$
$$-\frac{1}{36}\frac{N}{z^2}\frac{1}{kT}\Big[-(1-x_{B_l})^4\{(\omega_{aa_1}-\omega_{aa_2})^2$$
$$+(\omega_{aa_2}-\omega_{aa_3})^2+(\omega_{aa_3}-\omega_{aa_1})^2\}$$
$$+x_{B_l}(1-x_{B_l})^3\{(\omega^{l}_{aa'_1}-\omega^{l}_{aa'_2})^2$$
$$+(\omega^{l}_{aa'_2}-\omega^{l}_{aa'_3})^2+(\omega^{l}_{aa'_3}-\omega^{l}_{aa'_1})^2\}$$
$$+x^3{}_{B_l}(2-x_{B_l})\{(\omega_{a'a'_1}-\omega_{a'a'_2})^2$$
$$+(\omega_{a'a'_2}-\omega_{a'a'_3})^2+(\omega_{a'a'_3}-\omega_{a'a'_1})^2\}\Big]$$
$$(10)$$

where $N=N_A+N_{B_l}$.

For the system consisting of N_A molecules of A and N_{B_d} molecules of B_d, the chemical potential μ_{B_d} for the B_d molecule can be derived in quite the same way. Then $\mu^{S}{}_{B_d}$ becomes

$$\mu^{S}{}_{B_d}=kT\ln x_{dB}-\chi_{B_d}$$
$$+(1-x_{B_d})^2(2\chi_{AB_d}-\chi_A-\chi_{B_d})$$
$$-\frac{1}{36}\frac{N}{z^2}\frac{1}{kT}\Big[-(1-x_{B_d})^4\{\omega_{aa_1}-\omega_{aa_2})^2$$
$$+(\omega_{aa_2}-\omega_{aa_3})^2+(\omega_{aa_3}-\omega_{aa_1})^2\}$$
$$+x_{B_d}(1-x_{B_d})\,(^3\{(\omega^{d}_{aa'_1}-\omega^{d}_{aa'_2})^2$$
$$+(\omega^{d}_{aa'_2}-\omega^{d}_{aa'_3})^2+(\omega^{d}_{aa'_3}-\omega^{d}_{aa'_1})^2\}$$
$$+x^3{}_{B_d}(2-x_{B_d})\{(\omega_{a'a'_1}-\omega_{a'a'_2})^2$$
$$+(\omega_{a'a'_2}-\omega_{a'a'_3})^2+(\omega_{a'a'_3}-\omega_{a'a'_1})^2\}\Big]$$
$$(11)$$

where

$$\chi_{AB_d}=\frac{1}{z}\Big\{(z-1)^2\omega_{bb'}+(z-1)(\omega_{ab'}+\omega_{a'b})$$
$$+\frac{1}{3}(\omega^{d}_{aa'_1}+\omega^{d}_{aa'_2}+\omega_2{}^{d}_{aa'_3})\Big\} \qquad (12)$$

Now compare 10 with 11. Since B_l is a mirror image of B_d and B_l molecules are energetically equivalent, it follows that

$$\chi_{B_l}=\chi_{B_d} \qquad (13)$$

and from the relation

$$\omega^{l}_{aa'_1}+\omega^{l}_{aa'_2}+\omega^{l}_{aa'_3}=\omega_{11'}+\omega_{23'}+\omega_{32'}+\omega_{22'}$$
$$+\omega_{13'}+\omega_{31'}+\omega_{33'}+\omega_{12'}+\omega_{21'}$$
$$=\omega_{11'}+\omega_{22'}+\omega_{33'}+\omega_{13'}+\omega_{21'}$$
$$\omega_{32'}+\omega_{12'}+\omega_{23'}+\omega_{31'}$$
$$=\omega^{d}_{aa'_1}+\omega^{d}_{aa'_2}+\omega^{d}_{aa'_3}$$

it follows that

$$\chi_{AB_l}=\chi_{AB_d} \qquad (14)$$

Then the only difference between 10 and 11 is the second term in the bracket [ ] which contain the coefficient $x_B(1-x_B)^3$*.

By using new notations that $\chi_B=\chi_{B_l}=\chi_{B_d}$ and $\chi_{AB}=\chi_{AB_l}=\chi_{AB_d}$, and expressing mole fractions of B_l and B_d molecules in their saturated solutions by x_l, and x_d respectively, x_l and x_d are correlated by the following equations

$$\mu^{S}{}_{B_l}(x_l)=kT\ln x_l-\chi_B$$
$$+(1-x_l)^2(2\chi_{AB}-\chi_A-\chi_B)$$
$$-\frac{1}{36}\frac{N}{z^2}\frac{1}{kT}\{-(1-x_l)^4C_{AA}$$
$$+x_l(1-x_l)^3C_{AB_l}+x_l{}^3(2-x_l)C_{BB}\} \quad (15)$$
$$=\mu^{S}{}_{B_d}(x_d)=kT\ln x_d-\chi_B$$
$$+(1-x_d)^2(2\chi_{AB}-\chi_A-\chi_B)$$
$$-\frac{1}{36}\frac{N}{z^2}\frac{1}{kT}\{-(1-x_d)^4C_{AA}$$
$$+x_d(1-x_d)^3C_{AB_d}+x_d{}^3(2-x_d)C_{BB}\}$$
$$(16)$$

where

$$C_{AA}=(\omega_{aa_1}-\omega_{aa_2})^2+(\omega_{aa_2}-\omega_{aa_3})^2$$
$$+(\omega_{aa_3}-\omega_{aa_1})^2$$
$$C_{BB}=(\omega_{a'a'_1}-\omega_{a'a'_2})^2+(\omega_{a'a'_2}-\omega_{a'a'_3})^2$$
$$+(\omega_{a'a'_3}-\omega_{a'a'_1})^2$$
$$C_{AB_l}=(\omega^{l}_{aa'_1}-\omega^{l}_{aa'_2})^2+(\alpha^{l}_{aa'_2}-\omega^{l}_{aa'_3})^2$$
$$+(\omega^{l}_{aa'_3}-\omega^{l}_{aa'_1})^2$$
$$C_{AB_d}=(\omega^{d}_{aa'_1}-\omega^{d}_{aa'_2})^2+(\omega^{d}_{aa'_2}-\omega^{d}_{aa'_3})^2$$
$$+(\omega^{d}_{aa'_3}-\omega^{d}_{aa'_1})^2$$

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* x_B denotes χ_{B_l} or χ_{B_d}.

Discussions

The chemical potentials of B_l and B_d molecules in the solution $\mu^{S}{}_{B_l}(x)$ and $\mu^{S}{}_{B_d}(x)$ respectively, can be expressed as a function of their mole fraction x by the same formula except for the term containing C_{AB_l} and C_{AB_d}. If the common part is expressed by $F(x)$, the following formula is obtained.

$$\mu^{S}{}_{B_l}(x)=F(x)+\frac{1}{36}\frac{N}{z^2}\frac{1}{\boldsymbol{kT}}C_{AB_l}x(1-x)^3+\cdots\cdots$$

$$\mu^{S}{}_{B_d}(x)=F(x)+\frac{1}{36}\frac{N}{z^2}\frac{1}{\boldsymbol{kT}}C_{AB_d}x(1-x)^3+\cdots\cdots$$

This formula is represented schematically in Fig. 1. It is clear from the above expression that the higher the temperature the greater the difference between the two curves is. The straight line parallel to the x axis representing $\mu=\mu^{K}$ crosses the curves representing $\mu^{S}{}_{B_l}(x)$ and $\mu^{S}{}_{B_d}(x)$ at points P_l and P_d. Projections of P_l and P_d on χ axis represent x_l and x_d respectively. As these two curves generally do not coincide with each other, x_l and x_d have different values. This shows that the solubilities of *d*- and *l*-optical isomers in optical active solvent differ from each other and the magnitude of the difference is determined by the difference between C_{AB_l} and C_{AB_d}. It is clear from the above argument that in principle the optical resolution is possible using the difference of solubility of *d*- and *l*-optical isomers in optical active solvents and by choosing a proper kind of molecule of A, which makes the difference of C_{AB_l} and C_{AB_d} large, the difference between x_l and x_d can be made large enough to obtain an efficient optical resolution.

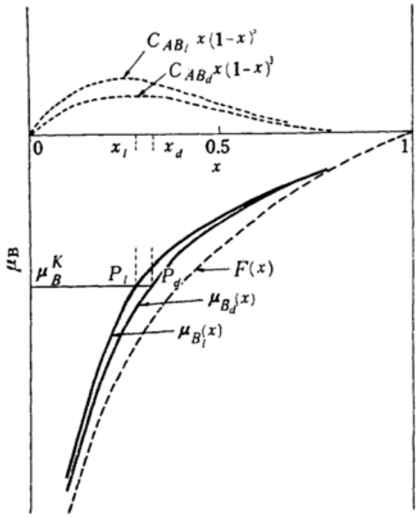

Fig. 1. Schematic diagram of μ_{B_l} and μ_{B_d} as a function of x.

The above argument is confined only to the mixture in which B_l and B_d molecules form no racemic compound. For the case where a racemic compound is formed more elaborate consideration must be made.

Summary

The statistical mechanical expression relating the solubilities of *d*- and *l*-optical isomers in optically active solvent is derived.

It is shown that there exists a difference between these two solubilities.

The possibility of optical resolution is also discussed.

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